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STRESS RISE DUE TO OFFSET WELDS IN TENSION

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ABSTRACT

If a butt weld in a sheet structure under tension is mismatched, an offset is produced which causes bending stresses at the weld in addition to the tensile stresses. The total stress field is calculated for the flat plate and also for a symmetrical mismatch in a cylindrical pressure vessel.

CONTENTS

ILLUSTRATIONS

STRESS RISE DUE TO OFFSET WELDS IN TENSION

In pressure vessels made of formed plate elements, there arises the possibility of local stress concentrations due to eccentricities caused by weld mismatch or plate overlap. These generally occur in girth welds and may give rise to relatively high local stresses which, in less ductile materials, may lead to early failures of the pressure vessel. Two limiting cases will be discussed in order to set bounds on the maximum stresses to be considered.

Case 1

In the first case, the pressure vessel is assumed to have a very large radius so that the plate elements can be assumed to be flat (the weld is assumed to be a girth or circumferential weld). Also, the eccentricity is assumed to be localized and only extends for a short distance around the periphery of the vessel. These assumptions lead to the consideration of a flat sheet specimen of unit width, containing an offset. The offset is located midway between fixed ends of the strip and the system is loaded in tension with a force P (see Figure 1).

The small element ABCD in Figure 1 is assumed to be rigid and to rotate as a solid body under the action of the two plate elements attached to it. We see therefore that the problem resolves itself into two parts, namely,

a. Moment and force equilibrium of the offset element ABCD

b. A beam column problem for the two sheet elements AG and CE, consisting of a cantilever beam with an end shear F, an axial tensile load P, and an end moment M_o.

If the offset is d and the width of the offset area is w then the equilibrium of the element is given by

$$2PR\cos(\alpha+\beta) - 2FR\sin(\alpha+\beta) - 2M_o = 0 \tag{1}$$

or

$$PR(\cos\beta\cos\alpha - \sin\beta\sin\alpha) - FR(\sin\alpha\cos\beta + \cos\alpha\sin\beta) - M_o = 0 \tag{2}$$

But

$$\sin\beta = \frac{w}{2R} \quad \text{and} \quad \cos\beta = \frac{d}{2R} \tag{2a}$$

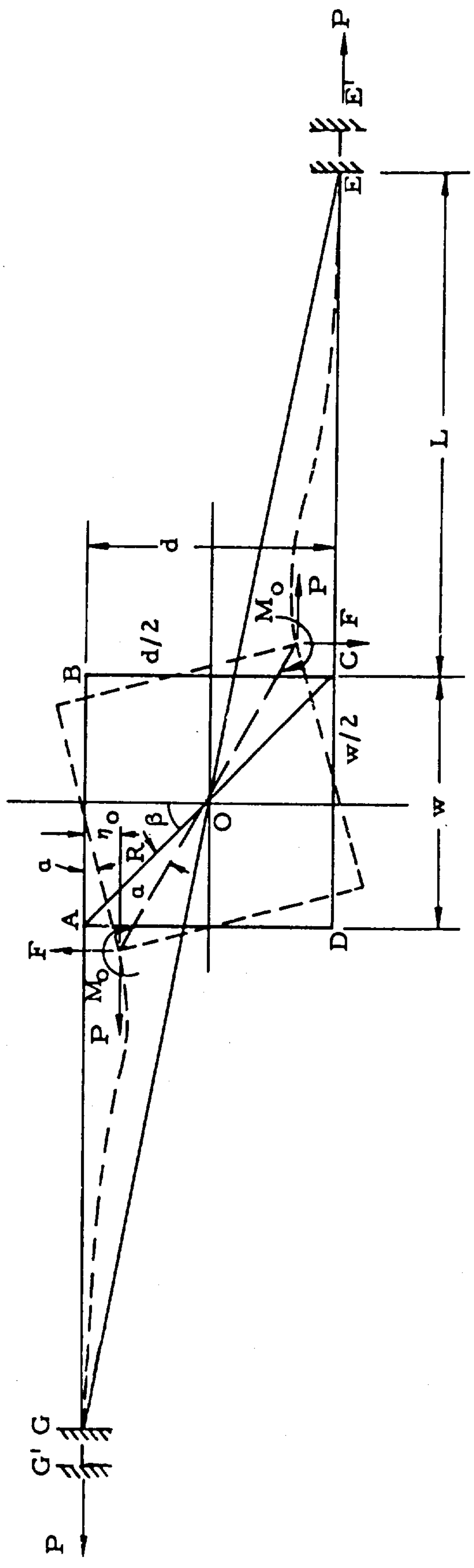

Figure 1. Forces and Moments on ABCD.
(1696)

Assuming a is small, we can write

$$\sin a \doteq a \qquad \cos a \doteq 1$$

and equation (1) becomes

$$P(d - wa) - F(w + da) - 2M_o = 0 \tag{3}$$

The second part of the problem is obtained by a consideration of the cantilever beam shown in Figure 2. The basic differential equation is

$$EI \frac{d^2\eta}{dx^2} = Fx - P(\eta_o - \eta) - M_o \qquad (+\ M = \circlearrowright) \tag{4}$$

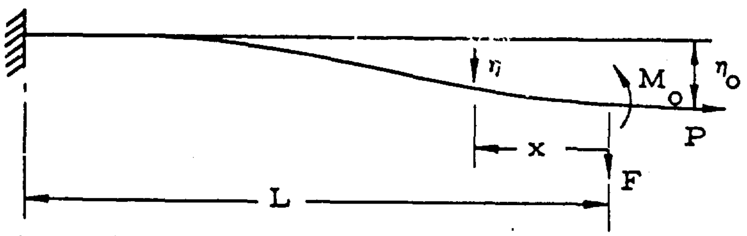

Figure 2. Assumed Beam Loading.
(1697)

This has a solution

$$\eta = A \sinh k_o x + B \cosh k_o x - \frac{Fx}{P} + \eta_o + \frac{M_o}{P} \tag{5}$$

where

$$k_o = \sqrt{\frac{P}{EI}} \tag{6}$$

Using the boundary conditions that $\eta = d\eta/dx = 0$ at $x = L$ and that $EI(d\eta^2/dx^2) = -M_o$ and $\eta = \eta_o$ at $x = 0$ we find that

$$\eta(x) = \left(\frac{F}{Pk_o \cosh \lambda_o} + \frac{M_o}{P} \tanh \lambda_o\right) \sinh k_o x - \frac{M_o}{P}(\cosh k_o x - 1) - \frac{Fx}{P}$$

$$+ \frac{F}{Pk_o}(\lambda_o - \tanh \lambda_o) - \frac{M_o}{P}(1 + \sinh \lambda_o \tanh \lambda_o - \cosh \lambda_o) \tag{7}$$

$$\eta(0) = \frac{F}{Pk_o}(\lambda_o - \tanh \lambda_o) - \frac{M_o}{P}(1 + \sinh \lambda_o \tanh \lambda_o - \cosh \lambda_o) \tag{8}$$

$$\theta(x) = \left(\frac{F}{P \cosh \lambda_o} + \frac{M_o k_o}{P} \tanh \lambda_o\right) \cosh k_o x - \frac{M_o k_o}{P} \sin k_o x - \frac{F}{P} \tag{9}$$

$$\theta(0) = \frac{F}{P} \left(\frac{1}{\cosh \lambda_o} - 1\right) + \frac{M_o k_o}{P} \tanh \lambda_o \tag{10}$$

where

$$\lambda_o = k_o L = L \sqrt{\frac{P}{EI}} \tag{11}$$

From Figure 1 we observe that

$$\eta(0) \doteq R\alpha \sin \beta = R\alpha \frac{w}{2R} = \frac{\alpha w}{2} \tag{12}$$

and

$$\alpha = \theta(0)$$

thus, the three definitive equations of the problem are

$$P(d - w\alpha) - F(w + d\alpha) - 2M_o = 0 \tag{13}$$

$$\frac{\alpha w}{2} = \frac{F}{Pk_o} (\lambda_o - \tanh \lambda_o) - \frac{M_o}{P} (1 + \sinh\lambda_o \tanh \lambda_o - \cosh \lambda_o) \tag{14}$$

$$\alpha = \frac{F}{P} \left(\frac{1 - \cosh \lambda_o}{\cosh \lambda_o}\right) + \frac{M_o k_o}{P} \tanh \lambda_o \tag{15}$$

which are to be solved for α, F, and M_o in terms of P. Since the most serious problems arise from high stresses (P large) and, in most practical cases, L is relatively large, then $\lambda_o = L \sqrt{P/EI}$ will be sufficiently large (> 4) so that the following approximations are justified:

$$\sinh \lambda_o \doteq \cosh \lambda_o \gg 1 \tag{16}$$

$$\tanh \lambda_o \doteq 1 \tag{17}$$

Equations (13), (14), and (15) then reduce to

$$P(d - w\alpha) - F(w + d\alpha) - 2M_o = 0 \tag{18}$$

$$\frac{\alpha w}{2} = \frac{F}{Pk_o} (\lambda_o - 1) - \frac{M_o}{P}$$

$$\alpha = -\frac{F}{P} + \frac{M_o k_o}{P} \tag{20}$$

Solving simultaneously we find that

$$F = M_o k_o \frac{A + 2}{A + 2\lambda_o - 2} \qquad (21)$$

$$a = \frac{2 M_o k_o}{P} \frac{\lambda_o - 2}{A + 2\lambda_o - 2} \qquad (22)$$

$$P d = M_o (A + 2) \left[1 + \frac{2 M_o d}{EI} \frac{\lambda_o - 2}{(A + 2\lambda_o - 2)^2} \right] \qquad (23)$$

where

$$A = w k_o = w \sqrt{P/EI} \qquad (24)$$

Detailed study indicates that the last term in equation (23) is negligible, therefore

$$M_o = \frac{P d}{A + 2} \qquad (25)$$

$$F = \frac{P d k_o}{A + 2\lambda_o - 2} \qquad (26)$$

$$a = \frac{2 k_o d (\lambda_o - 2)}{(A + 2)(A + 2\lambda_o - 2)} \qquad (27)$$

The maximum bending stress due to M_o is

$$\sigma_b = \pm \frac{M_o \, t/2}{t^3/12} = \pm \frac{6 M_o}{t^2} \qquad (28)$$

The direct stress is

$$\sigma_o = \frac{P}{A} = \frac{P}{t} \qquad (29)$$

and the maximum total stress is

$$\sigma_T = \sigma_o + \sigma_b = \frac{P}{t} + \frac{6 M_o}{t^2} \qquad (30)$$

the ratio of the total stress to the direct stress is therefore given by

$$\sigma_T/\sigma_o = 1 + \frac{6M_o}{Pt} = 1 + \frac{6 \; d/t}{A+2}$$

$$= 1 + \frac{3\delta}{\varepsilon\rho + 1} \tag{31}$$

where

$$\delta = d/t, \quad \varepsilon = w/t, \quad \rho = k_o t/2 \tag{32}$$

actually

$$\rho = \frac{k_o t}{2} = \frac{t}{2}\sqrt{\frac{P}{EI}} = \frac{t}{2}\sqrt{\frac{\sigma_o t}{Et^3/12(1-\mu^2)}}$$

$$= \sqrt{3(1-\mu^2)\,\sigma_o/E} \tag{33}$$

Equation (33) is shown plotted in Figure 3 and several examples are calculated and plotted in the Appendix.

<u>Case 2</u>

In this case, it is assumed that the discontinuity has axial symmetry with respect to a cylindrical axis and that the amount of offset is the same at all points. Also, the effect of radial support is taken into account (i.e., small radii are permissible). It is, however, assumed that the cylinder is long and approximates an infinite cylinder in both sides of the discontinuity (see Figure 4). In this case, the axial symmetry causes an elastic restraint to any deflection setting up an elastic support under the beam element under consideration. If the elastic support is k_s the reaction from any deformation η is given by

$$p = k_s \eta \tag{34}$$

Again assuming a unit width element, the problem is that shown in Figure 5, of a beam on an elastic support and loaded with an end moment and shear plus an axial load. From Reference 1 we find the equations for this case as follows.

[1]Hetenyi, Beams on Elastic Foundations.

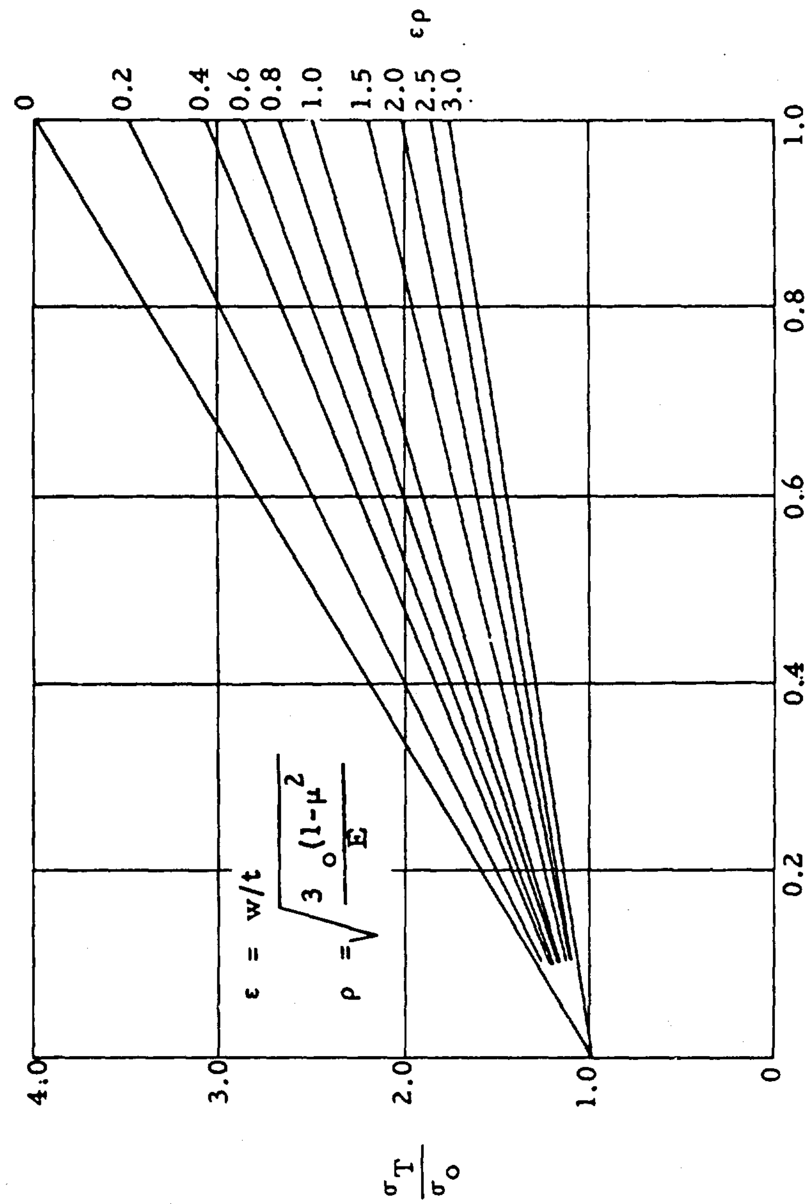

Figure 3. Ratio of Maximum Stress to Axial Stress for $\lambda_o = Lk_o \gg 4$.
(1698)

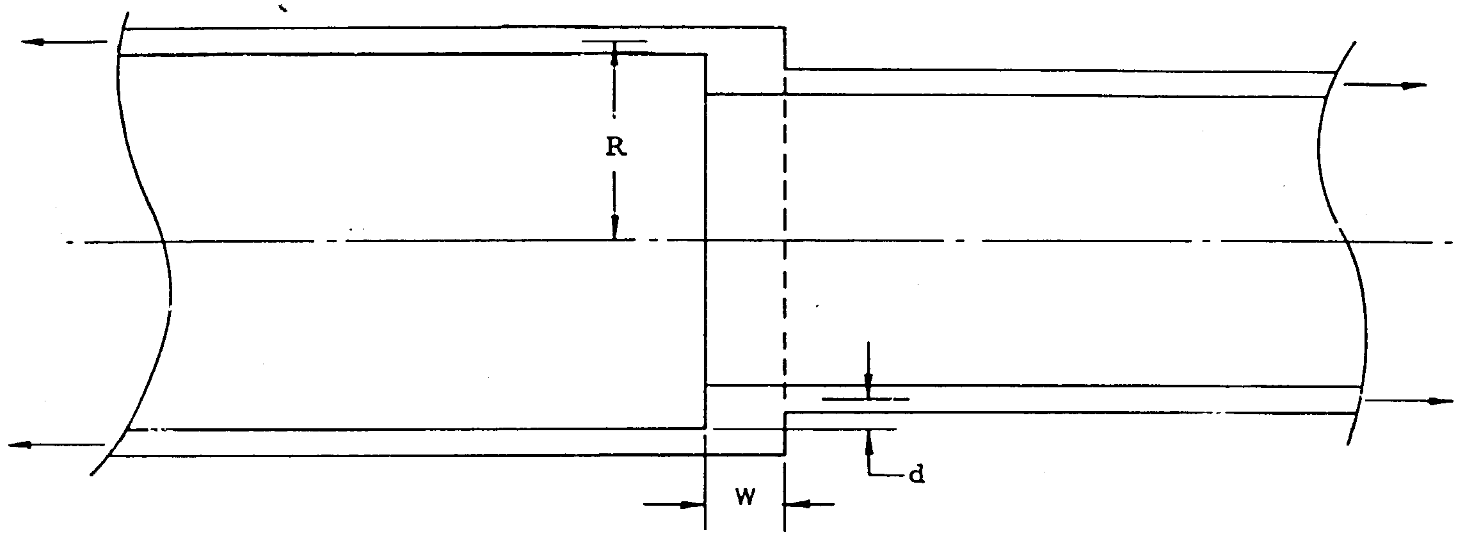

Figure 4. Weld Offset in Infinite Cylinder.
(1699)

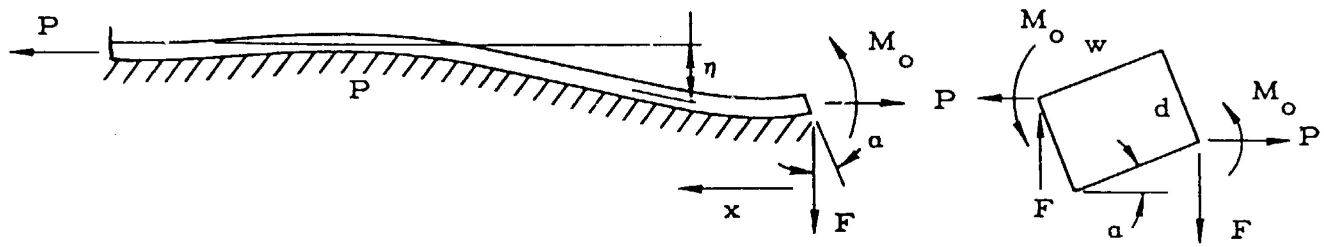

Figure 5. Free Body Diagram.
(1700)

Defining

$$\beta_1 = \sqrt{\sqrt{\frac{k_s}{4EI}} + \frac{P}{4EI}} = \sqrt{\lambda_s^2 + \frac{k_o^2}{4}} \tag{35}$$

$$\beta_2 = \sqrt{\frac{P}{4EI} - \sqrt{\frac{k_s}{4EI}}} = \sqrt{\frac{k_o^2}{4} - \lambda_s^2} \tag{36}$$

$$\lambda_s = \sqrt[4]{\frac{k_s}{4EI}} \tag{37}$$

The deflection (equations in this form are for $P > 2\sqrt{k_s EI}$ although it is found that the equations finally derived for $\eta(0)$ and $\theta(0)$, equations (39) and (42), are the same for all values of P).

$$\eta(x) = \frac{F}{\beta_2 k_s} \frac{2\lambda_s^2}{3\beta_1^2 + \beta_2^2} \left[2\beta_1\beta_2 \cosh \beta_2 x + (\beta_1^2 + \beta_2^2) \sinh \beta_2 x \right] e^{-\beta_1 x}$$

$$- \frac{M_o}{EI} \frac{1}{3\beta_1^2 + \beta_2^2} \frac{1}{\beta_2} \left[\beta_2 \cosh \beta_2 x - \beta_1 \sinh \beta_2 x \right] e^{-\beta_1 x} \tag{38}$$

from which

$$\eta(0) = \frac{k_o^2}{2\lambda_s^2 + k_o^2} \left\{ \frac{\beta_1}{\lambda_s^2} \frac{F}{P} - \frac{M_o}{P} \right\} = \frac{C}{P} \left\{ \frac{\beta_1}{\lambda_s^2} F - M_o \right\} \tag{39}$$

since

$$3\beta_1^2 + \beta_2^2 = 2\lambda_s^2 + k_o^2 \quad \text{and} \quad EI = P/k_o^2 \tag{40}$$

Similarly for the slope

$$\theta(x) = -\frac{F}{EI} \frac{1}{3\beta_1^2 + \beta_2^2} \frac{1}{\beta_2} (\beta_2 \cosh \beta_2 x + \beta_1 \sinh \beta_2 x) e^{-\beta_1 x}$$

$$+ \frac{M_o}{EI} \frac{1}{3\beta_p^2 + \beta_2^2} \frac{1}{\beta_2} \left[2\beta_1\beta_2 \cosh \beta_2 x - (\beta_1^2 + \beta_2^2) \sinh \beta_2 x \right] e^{-\beta_1 x} \tag{41}$$

which gives

$$\theta(0) = \frac{k_o^2}{2\lambda_s^2 + k_o^2}\left[-\frac{F}{P} + \frac{2M_o\beta_1}{P}\right] = \frac{C}{P}\left\{-F + 2M_o\beta_1\right\} \tag{42}$$

where

$$C = \frac{k_o^2}{2\lambda_s^2 + k_o^2} \tag{43}$$

The three definitive equations for this case are, therefore

$$P(d - w\alpha) - F(w + d\alpha) - 2M_o = 0 \tag{44}$$

$$\frac{\alpha w}{2} = \frac{C}{P}\left(\frac{\beta_1}{\lambda_s^2} F - M_o\right) \tag{45}$$

$$\alpha = \frac{C}{P}(-F + 2M_o\beta_1) \tag{46}$$

For the case of axial symmetry

$$k_s = \frac{Et}{R^2} \tag{47}$$

and

$$\lambda_s^2 = \sqrt{\frac{k_s}{4EI}} = \frac{\sqrt{3(1-\mu^2)}}{Rt} = \lambda_R/t^2 \tag{48}$$

where

$$\lambda_R = \sqrt{3(1-\mu^2)}\, t/R \tag{49}$$

From equation (32) $k_o = 2\rho/t$ and β_1 becomes

$$\beta_1 = \sqrt{\lambda_s^2 + \frac{k_o^2}{4}} = \frac{1}{t}\sqrt{\lambda_R + \rho^2} \tag{50}$$

Making the substitutions indicated, and solving equations (44), (45), and (46) simultaneously yields (by neglecting the $-F\,d\alpha$ term in equation (44), as in Case 1),

$$F = P\delta \frac{\lambda_R\left(1+\varepsilon\sqrt{\lambda_R+\rho^2}\right)}{2\varepsilon\left(\lambda_R+\rho^2\right)+\left(\varepsilon^2\lambda_R+2\right)\sqrt{\lambda_R+\rho^2}} \tag{51}$$

$$M_o = \frac{Pd}{2}\,\frac{\varepsilon\lambda_R+2\sqrt{\lambda_R+\rho^2}}{2\varepsilon\left(\lambda_R+\rho^2\right)+\left(\varepsilon^2\lambda_R+2\right)\sqrt{\lambda_R+\rho^2}} \tag{52}$$

$$\alpha = \frac{2\delta\rho^2}{2\varepsilon\left(\lambda_R+\rho^2\right)+\left(\varepsilon^2\lambda_R+2\right)\sqrt{\lambda_R+\rho^2}} \tag{53}$$

again, since $\sigma_{b_m} = \pm 6M_o/t^2$ and $\sigma_o = P/t$ we find for σ_T/σ_o

$$\sigma_T/\sigma_o = 1 + \frac{3\delta\left(\varepsilon\lambda_R+2\sqrt{\lambda_R+\rho^2}\right)}{2\varepsilon\left(\lambda_R+\rho^2\right)+\left(\varepsilon^2\lambda_R+2\right)\sqrt{\lambda_R+\rho^2}} \tag{54}$$

which reduces to equation (31) for $R\rightarrow\infty$ $\lambda_R\rightarrow 0$. Summarizing the variables in equation (54)

$$\delta = d/t \qquad \varepsilon = w/t \qquad \lambda_R = \sqrt{3(1-\mu^2)}\,t/R \qquad \rho = \sqrt{3(1-\mu^2)}\,\sigma_o/E$$

To show the effect of λ_R on the stress concentration, two examples have been calculated and plotted in Figures 6 and 7. These figures show that an R/t ratio of approximately 200 corresponds very closely to the limiting case of $R/t\rightarrow\infty$ and, therefore, it is only necessary to apply the correct equation (54) for R/t values less than approximately 200. For all other R/t values the simple equation (31) will give satisfactory results even for the axially symmetrical mismatch. For a local mismatch, the value of k_s is greatly reduced and here again, Case 1 more nearly represents the problem.

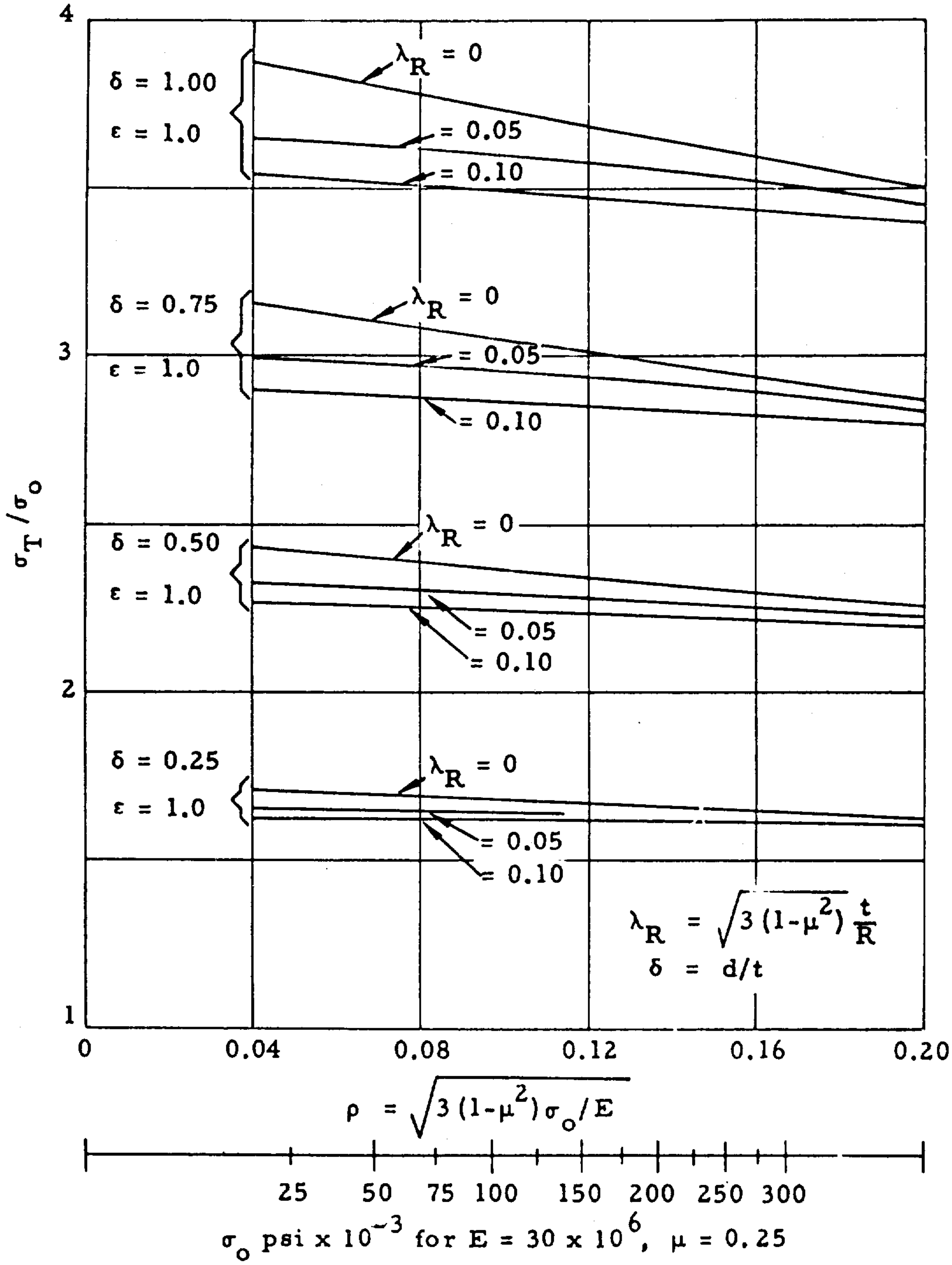

Figure 6. Effect of Variation of λ_R and δ for ε = w/t = 1.0.
(1701)

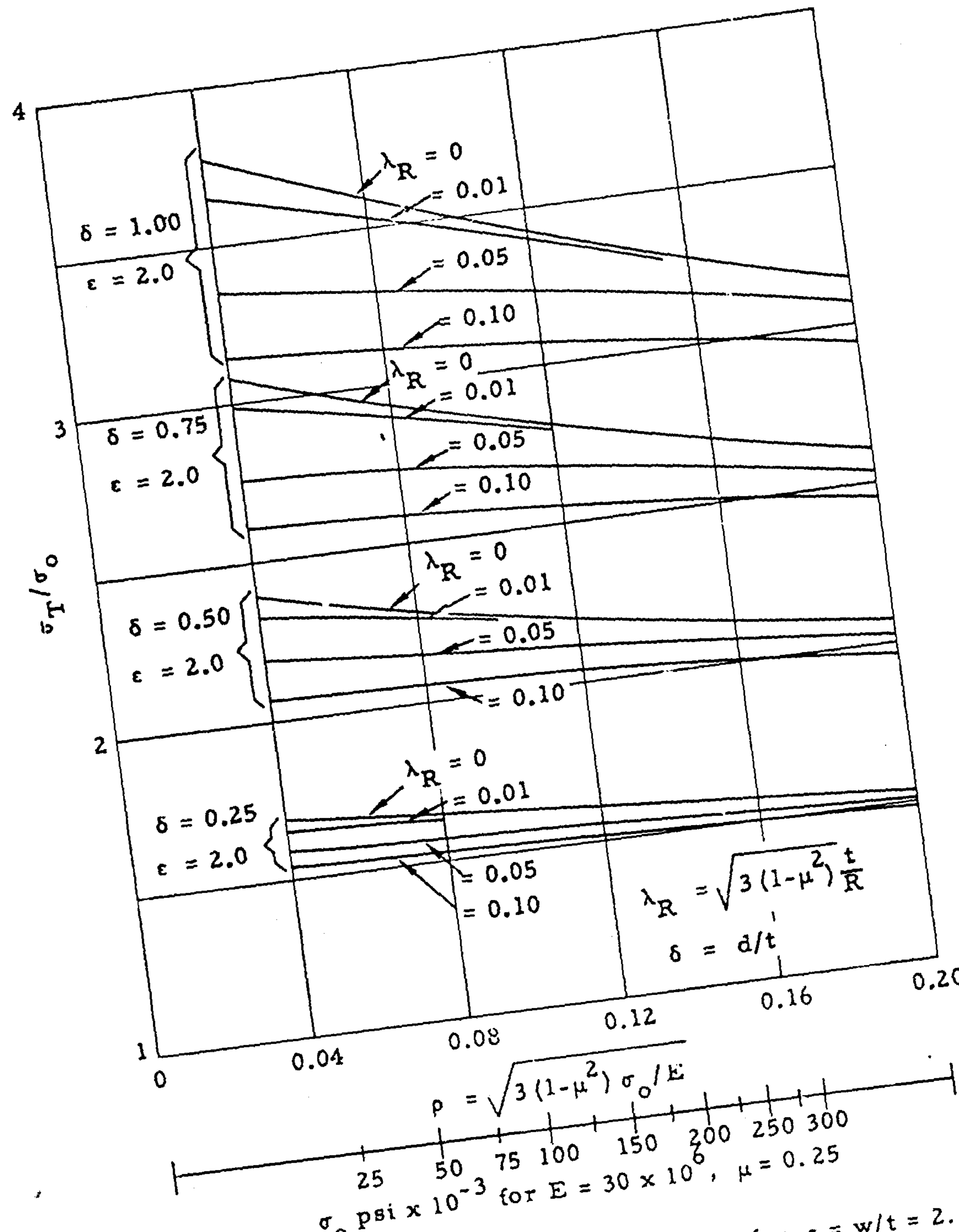

Figure 7. Effect of Variation of λ_R and δ for $\epsilon = w/t = 2.0$.
(1702)

DISCUSSION

Although the theory indicates relatively high stress concentrations due to even small offsets, the effect is highly localized. For example, the moment equation as derived from the deflection equation (38) shows that the moment decreases as $e^{-\beta_1 x}$. Since, for small λ_R, $\beta_1 \sim \rho/t$ [see equation (50)] we find that for $t = 0.10$ inch and $\sigma_o \doteq 100,000$ psi in steel, $\beta_1 \doteq 1$ and the moment reduces as e^{-x}. Thus, if the material has a reasonable amount of ductility for local deformation, the weld area will rotate due to surface yielding and failure will be avoided. On the other hand, if no local ductility exists, or the high stress region is further complicated by a sharp discontinuity at the weld line, local failure may be expected.

From a fatigue standpoint, where a pressure vessel is to be pressurized many times, weld offsets may have a serious effect since local yielding will not, in general, alleviate cyclic stress effects.

APPENDIX

TREND CURVES FOR TYPICAL WELD OFFSETS

Assumptions - Flat Plate Case - Equations (25), (26), (27)

$E = 30 \times 10^6$ psi. $t = 0.10$ in. $L = 4.0$ in.

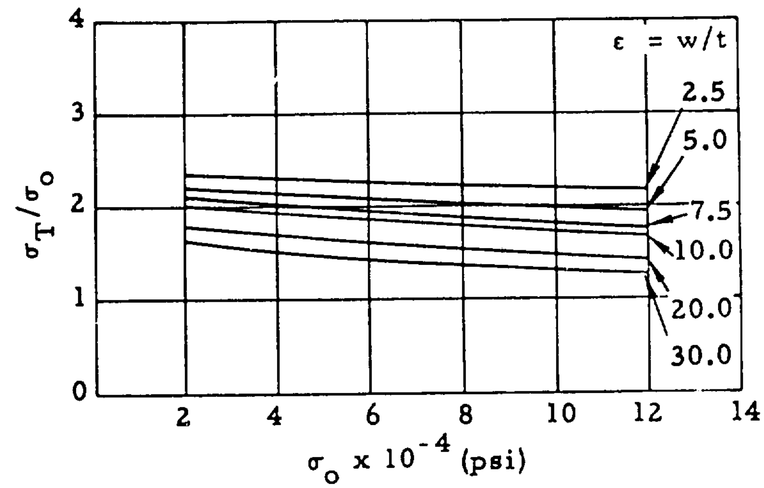

Figure A-1. Effect on σ_T/σ_o of Varying $\varepsilon = w/t$ with $\delta = d/t$ = constant = 0.5.
(1688)

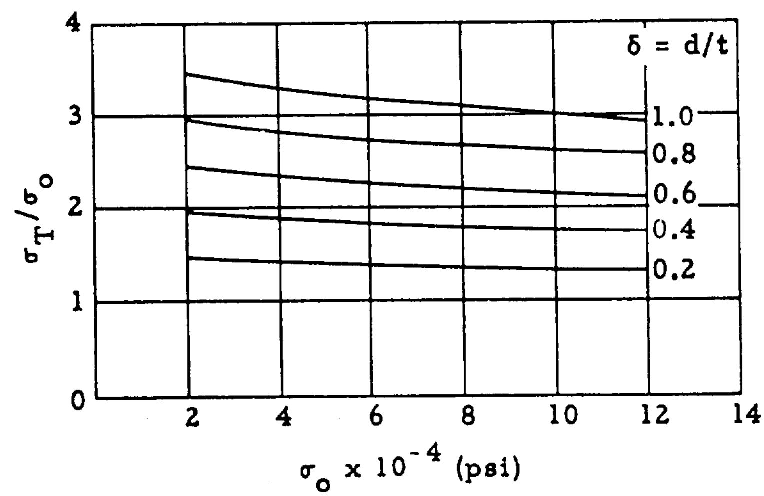

Figure A-2. Effect on σ_T/σ_o of Varying $\delta = d/t$ with $\varepsilon = w/t$ = constant = 5.0.
(1689)

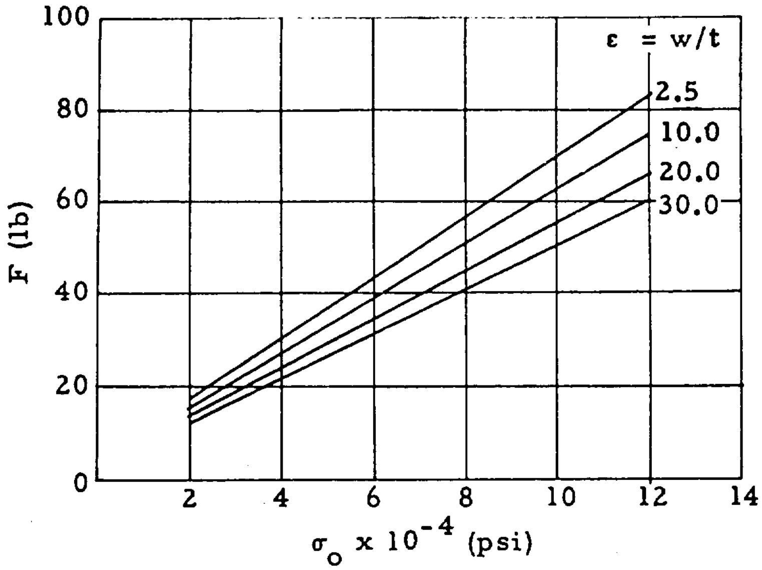

Figure A-3. Effect on F of Varying $\varepsilon = w/t$ with $\delta = d/t$ = constant = 0.05.
(1690)

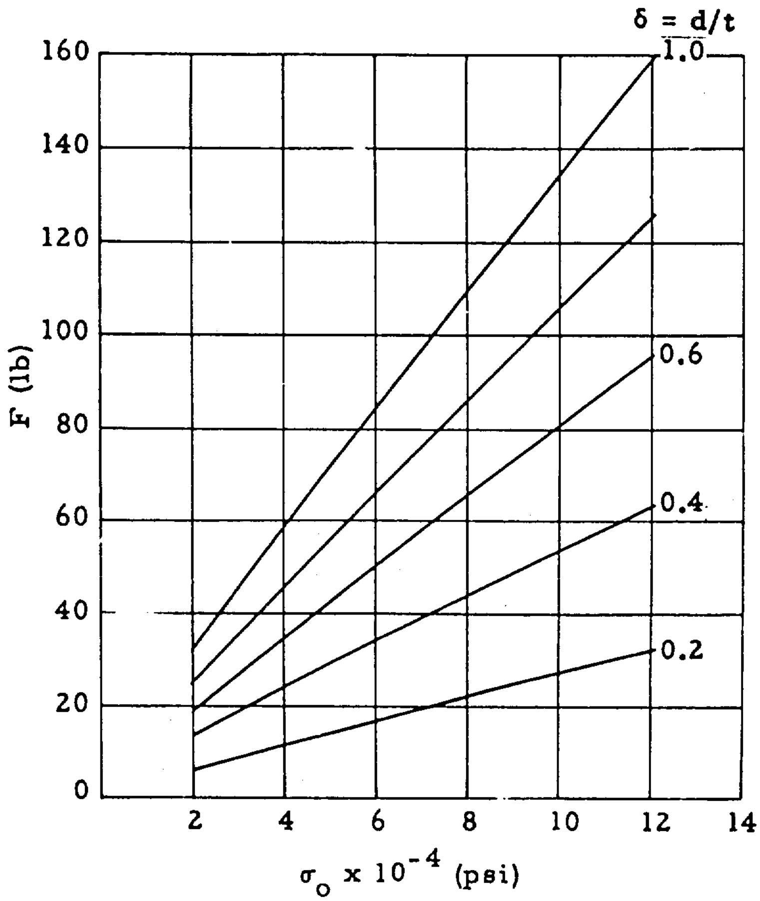

Figure A-4. Effect on F of Varying $\delta = d/t$ with $\varepsilon = w/t$ = constant = 5.0.
(1691)

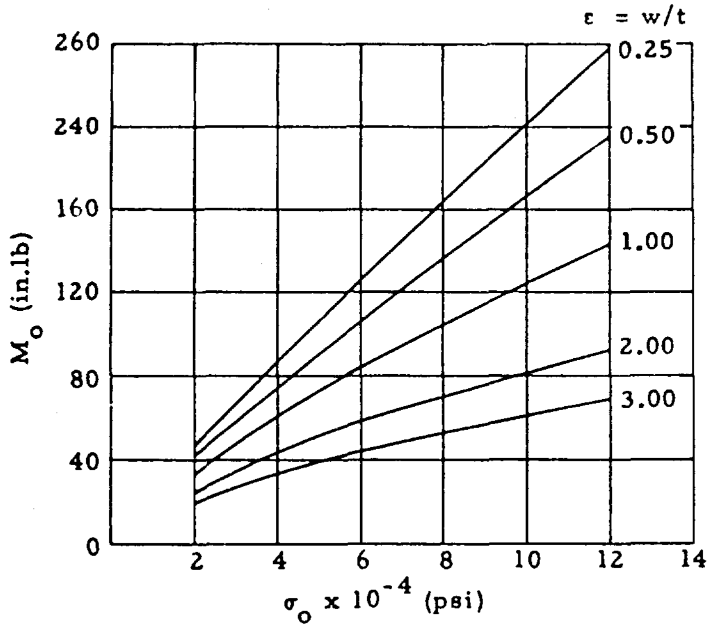

Figure A-5. Effect on M_o of Varying $\varepsilon = w/t$ with $\delta = d/t$ = constant = 0.5.
(1692)

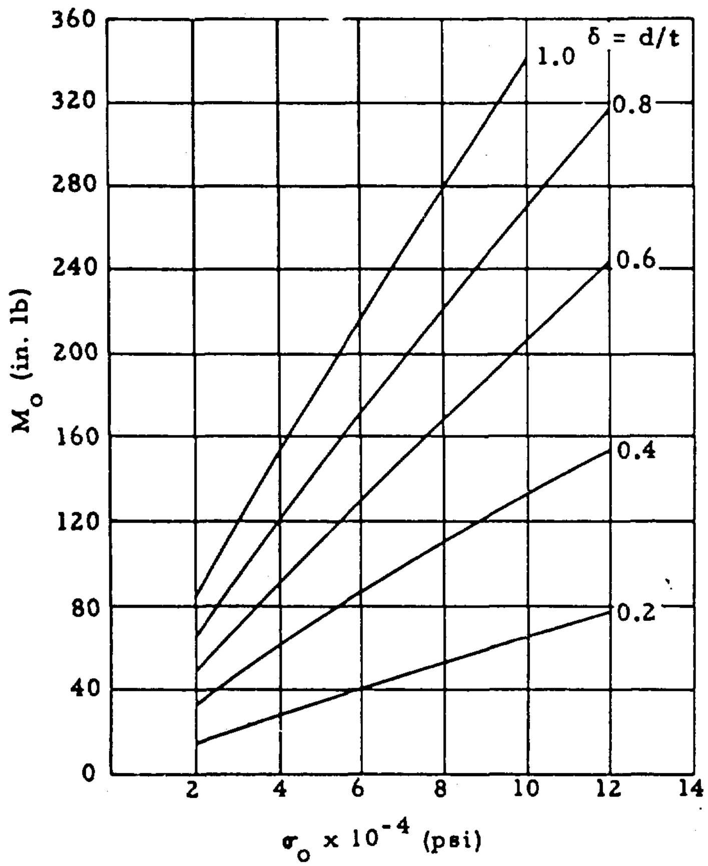

Figure A-6. Effect on M_o of Varying $\delta = d/t$ with $\varepsilon = w/t$ = constant = 5.0.
(1693)

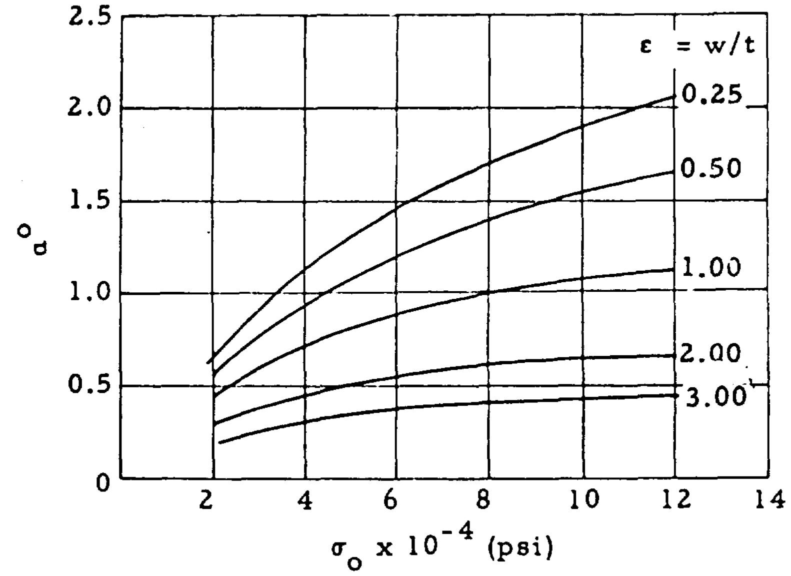

Figure A-7. Effect on α of Varying ϵ = w/t with δ = d/t = constant = 0.5.
(1694)

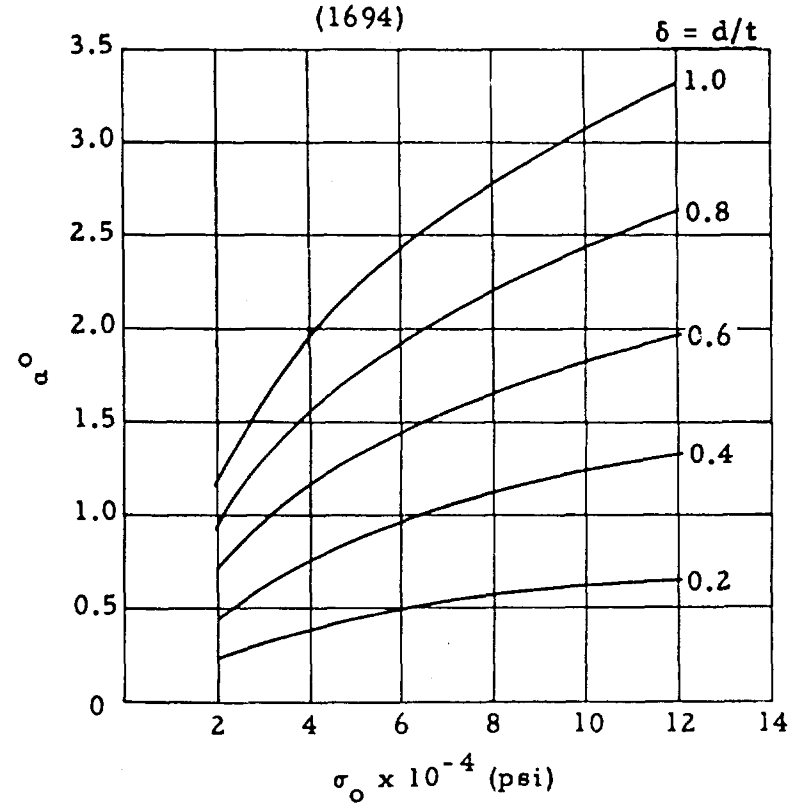

Figure A-8. Effect on α of Varying δ = d/t with ϵ = w/t = constant = 5.0.
(1695)